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Assessing Teachers' Knowledge in Analysing Errors in Mathematical Word Problems of Ghanaian Primary School Pupils

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Abstract

The survey investigated teacher knowledge in analysing pupils' wrong answer solutions in mathematical problem-solving. A sample of 205 pupils and teachers from 35 Islamic primary schools in the Greater Accra Region of Ghana were surveyed. The teachers were sampled through quota sampling, while the pupils were selected via stratified random sampling. A questionnaire and achievement test were used for the survey. Frequency count, percentage, and chi-square test *were used as statistical tools for data analysis. The study found that the majority (71.4%) of the primary school teachers in the Greater Accra Region had difficulty preparing a good marking scheme. Also, more than 60% of the teachers were unable to identify and analyse the errors of pupils as well as communicate feedback on the errors. The study found no statistically significant association between knowledge of error analysis and analysing wrong answer solutions (p>0.05). This study concluded that most primary school teachers in the Greater Accra Region of Ghana do not have enough knowledge in analysing mathematical wrong answer solutions of pupils by using Newman's model synthesis. It is recommended that mathematics teachers in Ghanaian basic schools should use Newman's model as a standard method of analysing pupils' work. For this reason, Ghana Education Service, Ghanaian universities, and colleges of education should include Newman's model during the training of mathematics teachers.*

Keywords: *Achievement test, Mathematical word problem, Newman's model, Wrong answer analysis.*

Introduction

In Ghana, the language demands of the mathematics curriculum are important and need to be developed by pupils. Pupils' ability to transform word problems into mathematical models is a skill that is crucial in mathematical information management. This is because by solving mathematics problems, pupils have the opportunity to demonstrate and develop certain cognitive abilities such as generalization, reasoning, data analysis, and the use of a variety of representations and strategies [1]. This brings to the fore pupils' understanding, comprehension skills, and analysis of real-life situations using mathematical models.

At the upper primary school levels, most errors in mathematics tests and examinations are caused by Reading, Comprehension, Transformation errors, or Carelessness [2]. This is as a result of the difficulties experienced with mathematics by pupils [3]. What explains why problem-solving activities are found in most mathematics textbooks? It is worth noting that word problem solving is not a topic by itself in the teaching syllabus, although nearly all topics include word problem solving as activities as indicated by the Ministry of Education [MoE] of Ghana [4].

It is well recognized that pupils struggle with both the literacy and mathematical demands of typical mathematical word problems. For this reason, many teachers and pupils seem to dislike

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word problems. Translating word problems into solvable equations and relations is one area of mathematics that pupils have a great deal of difficulty with (evident). The Chief Examiner's report from the West African Examination Council (WAEC) [5] revealed that pupils were unable to translate word problems into mathematical language and to use mathematical symbols to enable them to solve problems.

For example, pupils' solutions to the problem: Find the value of $x\ if\ \frac{3x-2}{5}$ is greater than $\frac{1-4x}{12}$ by 5 was varied in nature (WAEC Report, 2008).

Most of the candidates gave literal mathematical interpretation of the problem.

In writing the statement in symbols, they wrote:

$$\frac{3x-2}{5} > \frac{1-4x}{10}\ x\ 5$$

instead of

$$\frac{3x-2}{5} = \frac{1-4x}{10} + 5$$

which solves to obtain x = 5.5.

The mathematical wrong answer is an answer with either a wrong method (procedure) or incorrect final answer, or both (source). Wrong answers are a common phenomenon in pupils' mathematical problem-solving skills. School pupils, irrespective of their performance in mathematics, have once experienced wrong answers in mathematics [2]. Newman attributed the sources of errors to these factors, namely, reading, comprehension, transformation, application of processes skills, and encoding [6]. However, reading, comprehension, and transformation collectively contribute over 50% of the sources of working errors that pupils commit in solving word problems. More than 50% of the errors committed by the pupils in the course of solving mathematical word problems occur before the stage of process skills in Newman's hierarchy of errors. Apart from the five identified sources of errors, carelessness has also been noted to be another major source of error. Pupils may, at times, arrive at wrong solutions to problems not due to the lack of understanding of the problem or concept but due to carelessness. In Ghana, one of the reasons why pupils perform poorly in Mathematics is because most pupils lack proper feedback from teachers any time they wrongly solve a mathematical problem. There is enough evidence that suggests that teachers lack knowledge of wrong answer analysis [7].

Although educators observed that pupils are generally performing poorly in word problem-solving tasks, teachers are also consistently failing to rectify the underlining cause (Cross, 2008). Teachers fail to analyze and identify pupils' wrong answer solutions to mathematical problem-solving in other to provide remedy [8]. For this reason, suggestions have been made to assist teachers in identifying the sources of errors. A reflective assessment or analysis of the answer may thus reveal the source of error, leading to its correction or rectification. In this direction, pupils must be made to provide a written explanation or justification for the solution to the problem. Wrong answer analysis by teachers is a fundamental aspect of the teaching process. A teacher's understanding of pupils' wrong solutions to mathematical tasks guides the teacher in preparing lessons in mathematics and also for remedial teaching [7, 9]. People argue that teachers must give attention to wrong answers since this will increase teachers' knowledge of pupils' mathematical errors. Teachers should thus have a holistic view of what they teach in class. This process of having will enable teachers to investigate pupils' line of thinking systematically. Researchers [10,11] have shown that the analysis of the wrong answer solution method of teaching has the greatest impact on pupils' critical thinking. In their opinion, it gives the teacher the opportunity to analyze and identify the source of error and the thinking process of pupils [11]. Similarly, Berk et al. were of the view that wrong answer solution analysis provides teachers with the ability to identify the source of the error, why the errors are made and the thinking ability of their pupils [10].

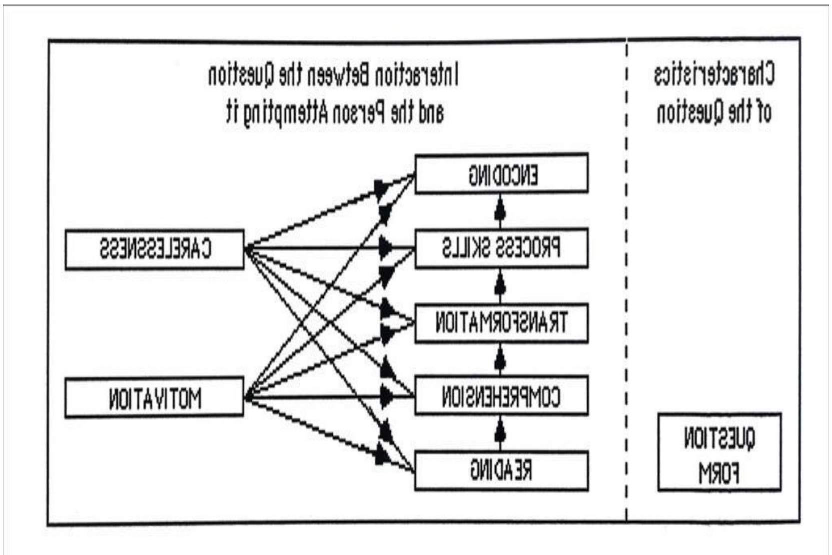

Figure 1. Hierarchy of Causes of Error as illustrated by Clements (1980)

Teacher knowledge in this paper is the ability to construct and use comprehensive marking schemes appropriately. Teachers' inadequate knowledge in analyzing pupils' wrong answers makes them to adopt the wrong approach to marking only the final answer arrived at by the pupils as right or wrong. Once they do not analyze the working process of the pupils, they are unable to detect the actual sources of errors. Educationists, teachers, policymakers, and researchers continually search for effective ways to improve the word problem-solving abilities of pupils in basic schools. Within wrong answer, analyses are embedded sequential and hierarchical errors, which can be described in a mathematical model proposed in 1983 by Newman [15]. In most cases, the teacher marks the answer wrong without considering pupils' thought processes and cognitive capabilities. This brings to light Newman's Model Synthesis of error identification and analysing in mathematics which is illustrated by Clements [12] in Figure 1.

According to Newman, a person wishing to obtain a correct solution to a one-step word problem must ultimately proceed according to some form of hierarchy [6]. To her, this hierarchy considers the issue of language as the first step in identifying the child's area of difficulty in word problem-solving. Newman suggested the following hierarchy was suggested: reading the problem; comprehending what is read; carrying out a mental transformation from the words of the question to the selection of an appropriate mathematical strategy (transformation); applying the process skills demanded by the selected strategy, and encoding the answer in an acceptable written form [6]. Newman used the word "hierarchy" because she reasoned that failure at any level of the above sequence prevents problem solvers from obtaining satisfactory solutions [13].

According to Clements, errors arising from the nature of a factor are essentially different from those in the other categories since the source of difficulty resides fundamentally in the question itself rather than in the interaction between the problem solver and the question [12].

Clements continued by stating that two other categories, "Carelessness" and "Motivation," may be demonstrated as separate from the hierarchy, although these types of errors can occur at any stage of the problem-solving process [12]. A careless error could be a reading error or a comprehension error. Similarly, someone who had read, comprehended, and worked out an appropriate strategy for solving a problem might decline to proceed further in the hierarchy because of a lack of interest.

Casey pointed out that problem solvers often return to lower stages of the hierarchy when attempting to solve problems [14]. Casey further indicated that in the middle of a complicated calculation, a child might decide to re-read the question to check whether all relevant information has been taken into account [14]. However, even if some of the steps were re-read during the word problem-solving process, Newman's hierarchy provides a fundamental framework for the sequencing of essential steps [6]. Along with the hierarchy, Newman recommended that an interview of pupils should be carried out in order to classify pupils' errors on written mathematical tasks [15]. In view of that, Newman suggested the following interview guide: the prescription is as follows: a) Please read the question to me. (*Reading*); b) Tell me what the question is asking you to do (*Comprehension*); c) Tell me a method you can use to find an answer to the question *(Transformation);* d) Show me how you worked out the answer to the question. Explain to me what you are doing as you do it *(Process Skills)*; e) Now write down your answer to the question *(Encoding)* [6]. If pupils who originally got a question wrong get it right when asked by an interviewer to do it once again, the interviewer should still make the five requests in order to obtain information on whether the original error could be attributed to carelessness or motivational factors.

According to Lemos, there is more empirical evidence on the impact of Newman's model synthesis in evaluating pupils' performance in the international literature, and none whatsoever for developing countries [16]. The international literature mainly utilizes the synthesis to appraise performance perhaps, because little research has been carried out in developing countries. Although Newman's hierarchy was helpful for the teacher, it could conflict with an educator's aspiration that the learner ought to experience his/her own capability by developing his/her own methods and ways [17]. The demonstration is thus carried out on the bases that there is no conflict. This is because the Newman's hierarchy cannot be considered a learning hierarchy in the strict sense of the expression by Kaphesi who explained that Newman's framework for the analysis of errors was not put forward as a rigid information processing model of problem solving [18]. According to him, the framework was meant to complement rather than to challenge descriptions of problem-solving processes such as those offered by [19]. With the Newman's approach, the teacher is attempting to stand back and observe an individual's word problem-solving efforts from a coordinated perspective; Polya on the other hand, was most interested in elaborating the richness of what Newman termed Comprehension and Transformation [19].

In most Ghanaian classrooms, scoring in mathematics by teachers is by "winner takes all", rather than by the "partial credit scale". Pupils' responses to questions or exercises are simply marked either right or wrong without considering the solution path. The Ghana Education Service - Science, Technology, and Mathematics Education, in collaboration with Japanese International Corporation Agency [GES-STME-JICA], at a workshop for teachers, observed that pupils' responses to questions or exercises are simply marked with a "cross" (X) for a wrong answer, or a "tick" (√) for a right answer [20]. This method of marking does not really indicate what a pupil did wrong or present a counter suggestion to a pupil's line of thinking. It is against this backdrop that this paper sought to investigate mathematics teachers' knowledge in applying Newman's model of analysing errors in mathematical word problems of Ghanaian primary school pupils in the Greater Accra Region.

Materials and Methods

This study adopted the cross-sectional survey design. The target population for this study was 42 head teachers, 252 class teachers, and 7083 pupils from 42 Islamic primary schools in the Greater Accra Region of Ghana. The Islamic primary schools in the Greater Accra Region were purposively selected for the study because it is among this group of schools that the problem was discovered during a workshop in Science, Technology, and Mathematics Education. A random sampling technique was used in selecting 35 schools out of the 42 Islamic primary schools.

Sample of 205 made up of 105 primary school pupils, and 105 teachers were selected from 35 out of 42 Islamic primary schools. The 105 teachers made up of 70 females, and 35 males were sampled through quota sampling technique. Stratified random sampling was also used to select 100 pupils for the study. Questionnaire of Cronbach's alpha coefficient (α) of 0.70 was used for the survey. Achievement test was also used to generate data for the study. The achievement test was diagnostic in nature and was used to assess teachers' level of knowledge of wrong answer analysis and common errors that pupils commit. To generate the achievement test, 100 class five pupils were randomly selected from five schools. The achievement test was based on the basic five mathematics syllabus. Test papers of four pupils were randomly selected and given to each teacher to prepare a marking scheme, mark, and score. They were also asked to identify pupils' error(s) (if any), suggest reason(s) for the errors identified, and offer remedies for correcting the error. The Statistical Package for Social Sciences version 26 was used to analyse the data using descriptive statistics (frequency count, percentage) and inferential statistics (chi-square test).

Results

Table 1. Teacher Preparation of Marking Scheme (n = 105)

Marks	**No. of teachers**	**Percentage**
1-3	23	21.9
4-6	34	32.4
7-9	18	17.1
10-12	22	21.0
13-15	8	7.6
Total	105	100

In Table 1, the item 'Marks', numbering from 1 to 15, were the marks scored by teachers. From the result, 23 (21.9%) teachers scored between 1 and 3 out of 15 marks, 34(32.4%) scored between 4 and 6 marks, 18 (17.1%) scored between 7 and 9 marks, while 22(21%) teachers scored between 10 and 12 marks. Only 8(7.6%) respondents scored between 13 and 15 marks. This result is an indication that the majority (71.4%) of the Islamic primary school teachers in the Greater Accra Region of Ghana had difficulty preparing a good marking scheme for their class tests. Respondents used the prepared marking scheme to mark the sampled class test of pupils. This is presented in Table 2.

Table 2. Marking of Class Test of Pupils by Teachers (n = 105)

Marks	**No. of teachers**	**Percentage**
1-3	19	18.1
4-6	29	27.6
7-9	34	32.4
10-12	18	17.1
13-15	5	4.8
Total	105	100

In Table 2, 5(4.8%) teachers scored between 13 and 15 marks, while 18(17.1%) scored 10-12 marks. The majority (82), representing 78.1% of teachers, scored below 10 out 15 marks. From the analysis, it could be inferred that even teachers who prepared a good marking scheme could not adhere to it in their marking and scoring. Since respondents could not prepare a good marking scheme, they could therefore not score the test well. It was observed that marks were allotted indiscriminately.

For some respondents, the total marks scored for a question was more than the marks allotted for the question.

Table 3. Teachers' Knowledge of Error Analysis (n = 105)

Statement	**Response**			
	Yes		**No**	
	Freq.	**%**	**Freq.**	**%**
Make an attempt to identify errors	35	33.3	70	66.7
Able to identify errors	28	26.7	77	73.3
Able to analyse errors	26	24.8	79	75.2
Able to understand pupils' thinking	16	15.2	89	84.8
Communicate error to pupils	21	20.0	84	80.0
Check on feedback	9	8.6	96	91. 4

In Table 3, 70 (66.7%) teachers averred that they did not make any attempt to analyse pupils' wrong answers when marking. Also, 77(73.3%) teachers were unable to identify the errors of pupils. Similarly, 79(75.2%) teachers were not able to analyse the errors of pupils. Again, 89(84.4%) teachers were unable to understand the thinking process of pupils in giving wrong answers. The majority (84), which represents 80% of the teachers, did not communicate the error to pupils, and 96(91.4%) teachers failed to check on feedback as to whether pupil understood their errors. It is inferred from the results in Table 3 that more than 60% of the teachers were unable to identify and analyse errors of pupils, as well as communicate feedback on the errors. This result suggests that the majority of the Islamic primary school teachers in the Greater Accra Region of Ghana have inadequate knowledge of error identification and analysis.

Table 4. Chi-square Analysis of the Association between Knowledge of Error Analysis and Analysing Wrong Answer Solution (n = 105)

Test	**df**	**X^2 value**	**p-value**
Pearson Chi-Square	5	17.25	0.10
Likelihood Ratio Chi Square (X^2)	5	19.009	<0.0679

Note. $X2$ test is significant at p<0.05 level (2-tailed)

In Table 4, a chi-square (X^2) test of independence was performed to examine the relationship between teacher's knowledge of error analysis and analysing wrong answer solutions.

The X^2 test showed no significant association between knowledge of error analysis and analysing wrong answer solution, X^2 (5, N = 105) = 17.25, p = 0.10. The p-value of 0.10 is more than 0.05 alpha (p=0.10>0.05). Hence, we fail to reject the null hypothesis and conclude that there is no association between knowledge of error analysis and analysing wrong answer solution. Therefore, there is no significant relationship between the two variables.

Discussions

The study found that the majority (71.4%) of the Islamic primary school teachers in the Greater Accra Region of Ghana had difficulty preparing a good marking scheme underlying concepts of a topic for class tests. The marking scheme provides all the important concepts and processes in arriving at an answer. Failure to prepare a good marking scheme implies errors will be committed in providing a solution to wrong answers solution. Marking schemes usually provide a systematic methodology in arriving at an answer, thereby making it easy to identify the source of error or misunderstanding of concepts. The marking of scripts largely depends on the type of marking scheme being used. Again, teachers were not able to use their own marking scheme appropriately. The marking of the scripts was not done according to the marking scheme provided; therefore, it become difficult to identify to analyze the wrong answer solution of pupils. Some teachers have the knowledge to prepare marking scheme, marking of scripts, and analyzing wrong answers solutions but only fail to carry it out because of class size. This agrees with Noraini, who observed that the class size of teachers, if not managed, can affect their concentration in analyzing wrong answers from pupils [21].

It came to light that the majority (78.1%) of the teachers had difficulty in marking class tests of pupils. The results of the study also revealed that teachers who prepared a good marking scheme could not adhere to it in their marking and scoring. Since most of the teachers could not prepare a good marking scheme, they could therefore not score class tests appropriately. The implication is that teachers will be tempted to allot marks indiscriminately. This could lead to a situation where the total marks scored for a question might be more than the marks allotted for the question. Evidence gathered from the study indicated that there is no statistically significant association between knowledge of error analysis and analysing wrong answer solution ($p>0.05$).

Also, findings from this research have shown that, more than 60% of the teachers were unable to identify and analyse errors of pupils as well as communicate feedback on the errors. This result suggests that the teachers have inadequate knowledge of error identification and analysis. In other words, primary school teachers do not have enough knowledge in analysing wrong answer solutions by pupils. This suggests that teachers lack the requisite knowledge in analysing the wrong answer solution of pupils. Most teachers are just interested in marking the final solutions of pupils without systematically analysing the solution methods of pupils. This implies that teachers’ extent of knowledge is not enough to appreciate pupils’ line of thinking and to identify their sources errors in solving mathematical problems. According to the findings teachers are not able to adequately identify and analyse the errors of pupils, although they have some little knowledge of analysing wrong answer solution. It is probable that the teachers do not have specialized knowledge and pedagogy in mathematics to understand the thinking process of pupils in the methodology in arriving at an answer. This finding validates the view of other researchers [7, 22, 23] who observed that because some teachers lack the technical understanding of concepts they do not even make an attempt to analyse the source of the error or identify the error in their opinion, teachers who attempt to analyse and identify source of error also fail because they do not have adequate knowledge of the Newman’s error synthesis model to adequately identify source of error.

Conclusions

The study concludes that the majority (over 60%) of Islamic primary school teachers in the Greater Accra Region of Ghana do not have enough knowledge in analysing mathematical wrong answer solutions of pupils by using Newman’s model synthesis. They fail to make an attempt in identify and analyse errors, understand pupils thinking, communicate errors and check on pupils for corrections or remedies. Based on the findings of this research, it is recommended that mathematics teachers in Ghanaian basic schools should use models Newman's model as a standard method of analysing pupil's work. For this reason, the National Council for Curriculum and Assessment (NaCCA) of the Ministry of Education, Ghana Education Service, Ghanaian universities, colleges of education, and curriculum developers should include Newman's model during the training of mathematics teachers. The introduction of Newman's model synthesis is likely to put teachers in a better position to effectively analyse pupils wrong answer solutions since they do not have any laid down procedure in doing so. Also, Ghanaian basic school mathematics teachers should try to develop students' arithmetic understanding first before advancing to higher levels of story problems. Since wrong answer analysis is beneficial to both teachers and pupils in the teaching and learning process, teachers should be encouraged to integrate it into their teaching process and to also encourage students to be more careful in problem solutions.

Conflict of Interest

The authors have no conflicts of interest to declare.

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