

Project Task – Resource Optimization using Eigen Vector Analysis: A Case Study of a 3-Bedroom Bungalow Construction

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Abstract

Construction projects involve complex task-resource interactions, making optimization crucial for timely completion and cost-effectiveness. This study applies eigenvector analysis to optimize task-resource allocation in construction projects, using a 3-bedroom bungalow case study. A matrix was developed representing tasks to be performed against resource utilization and eigenvector centrality measures applied to identify critical resources and tasks. The analysis revealed key resources driving the project timelines, enabling efficient allocation and minimizing delays. By prioritizing tasks and resources based on their centrality scores, project managers can streamline workflows, reduce idle time, and enhance productivity. The results demonstrate significant improvements in resource utilization, project duration, and overall efficiency. This approach provides valuable insights for construction project managers, enabling data-driven decision-making and optimized resource allocation. The study contributes to the growing body of research on applying network analysis techniques to construction project management, offering a practical tool for improving project outcomes. By leveraging eigenvector analysis, construction professionals can better navigate complex project dynamics, mitigate risks, and deliver projects on time and within budget.

Keywords: Allocation, Construction Project, Eigen Vector, Optimization, Task-Resource.

Introduction

Construction projects involve complex interactions between tasks and resources, leading to delays, cost overruns, and reduced productivity. efficient task-resource allocation [1] is crucial to ensure timely completion, cost-effectiveness, and quality. Various methods over the years have been proposed to address the task-resource optimization. These includes but not limited to the Critical Path Method (CPM) [2], the Program Evaluation and Review Technique (PERT), Resource Allocation and Multi-Project Scheduling (RAMPS). While these methods have shown promising results, eigen vector analysis offers a unique approach to identifying critical resources and tasks, enabling efficient allocation of both material

and human resources and minimizing project delays. The study applies eigen vector analysis to optimize task-resource allocation in construction projects using a 3-bedroom bungalow case study with the aim of identifying the critical resources and tasks to streamline workflows and reduce project duration. The cardinal objective is to design and develop a task – resource network model using a rectangular array of numbers, otherwise known as matrices, apply eigen value analysis and evaluate the corresponding eigen vectors to determine critical resources of the project. These will go a long way to assist in the task-resource allocation and also help in the evaluation and optimization of the project duration and its productivity. Analysis of

resource quantities using eigen vector analysis will also lead to the determination of critical resources necessary for optimal completion of the project. Even though, there are limitations of the study due to the fact that the model assumes a linear [3] relations between tasks, this study offers a novel approach to task-resource optimization by leveraging network analysis techniques to provide a practical tool for improving the project outcome.

Materials and Methods

The construction site was prepared by clearing the site, a surveyor was then brought to examine the suitability of the site for the construction of the 3 -bedroom bungalow. With all the soil and terrain test and analysis conducted, an architect was then made to draw the building plan. The construction of the main structure began when a professional mason was engaged to enable the realization of the final deliverable, the structure. When the construction began, it was pertinent to effectively manage the construction process using construction project management principles and theories to ensure that the final deliverable meet stakeholder's expectations by being of acceptable standards. This therefore called for the formation of stakeholders and the application material gathering techniques. These were necessary to enable the continuous monitoring and evaluation at each stage of the construction process to prevent the project from slipping through the cracks. In the material and human resource gathering phase, theoretical concepts were applied to determine the critical resources.

The Concept of Resource Optimization in Construction Projects

Resource optimization in construction projects involves the allocation of resources such as labor, materials, equipment, and time efficiently to minimize costs, reduce waste, and maximize productivity. This can be achieved with techniques such as;

1. Critical Path Method (CPM) [4] which is a step-by-step project management technique for project process planning that defines the critical and non-critical tasks with the aim of preventing time frame problems and project process bottlenecks. It is always constructed with the support of a network diagram which gives a visual representation of tasks and dependencies. Time durations are then estimated and assigned to the tasks with dependencies showing the relationship between the tasks. The critical path method helps the project manager to identify critical tasks, allocate resources effectively and ensure timely project completion.
2. Program evaluation and review technique (PERT) [5] which is a project management technique used to plan, schedule, and control projects by analyzing tasks, dependencies and timelines using the network diagram with critical path. The optimistic, pessimistic and most likely time estimates are often performed and the critical path necessary for minimum project duration estimated.
3. Simulation modelling which is a technique used to analyze and optimize complex systems by mimicking their behavior under various scenarios. In construction resource allocation, It helps project managers evaluate different resource allocation strategies, identify potential bottlenecks, and optimize resource utilization. It is often done by gathering the project data about resources with tasks and a simulated model built using a specialized software. The model is then simulated, optimal resource parameters determined and decisions taken. By leveraging simulation modeling, construction project managers can make informed decisions, optimize resource allocation, and improve project outcomes
4. Linear Programming [6] which also works by minimizing or maximizing an objective function subject to some constraints. It is

achieved by creating a mathematical representation of the goal followed by decision variables and finally, the constraints.

Resource optimization is crucial in construction projects to ensure efficient use of resources, minimize costs, and deliver projects on time.

The Concept of Eigen Values and Eigen Vectors

In this area of the construction management phase, the concept of eigen values and vectors and their application to task – resource optimization was used. This is because a quantitative research study applies numerical computations to analyze the available data at hand. The concept of eigen values and vectors applies linear programming techniques with the assumption that the tasks are linearly dependent on each other.

Eigen vectors and Eigenvalues are the vector and scalar quantities that are connected with the matrices which are used for linear transformations or linear operator [7]. A vector that is transformed by a scalar factor is referred to as an eigenvector, and the value of the scalar attached to the eigenvector is known as the eigenvalue.

An eigenvector of a rectangular matrix A is a vector x such that, the product of x and A on the left, produces a multiple of x.

That is:

$$Ax = \lambda x \quad (1)$$

$$Ax = \lambda x$$

λ = eigenvalue of A corresponding to the eigenvector x

To determine the eigenvalues, equation (1) is re-written as;

$$Ax - \lambda x = 0 \quad (2)$$

$$(A - \lambda)x = 0 \quad (3)$$

By introducing the identity Matrix, I,

$$(A - \lambda I)x = 0 \quad (4)$$

where I = the identity matrix.

From equation (4), Either;

$$x = 0 \text{ or } |A - \lambda I| = 0$$

But as $x \neq 0$, since it gives no information,

$$|A - \lambda I| = 0.$$

The λ is called an eigenvalue of the matrix A and x is called an eigenvector of A associated with λ , the set of all eigenvalues of the matrix A. Evaluating the determinant of the 6×6 matrix $(A - \lambda I)$, denoted as $|A - \lambda I|$ and equating the results to zero (0), will give a characteristic equation which is a polynomial involving λ with a power of six (6). This polynomial is impossible to compute manually. Hence, the use of MATLAB.

The eigen vectors of the matrix A, are the vectors x when multiplied by A results in a vector having the same or opposite direction to x.

Whiles eigen values represent scalar changes in linear transformations and capable of analyzing and giving an indication of a system stability, convergence or divergence, eigen vectors represents directional linear transformations [8] with a scale factor. Hence, they can be applied in a matrix dimensionality reduction. When applied in task- resource optimization in a construction project, they can assist in the efficient allocation of resources and the vector analysis will show the critical areas of the project where resources will be channeled to enable a successful completion.

Application of Eigen Vectors in the Optimization of Construction Data

During construction projects, it is always required to carry out the project in a very successful manner. This can only be achieved if the project resources in terms of humans and materials are well optimized whiles ensuring stakeholder satisfaction.

Eigenvalues and eigen vectors can help optimize resource distribution, identifying key tasks and resources. Their analysis can determine critical path tasks, ensuring timely

project completion. They also help identify sensitive tasks and resources [9], enabling proactive risk mitigation. On the overall, eigen vectors and eigenvalues can optimize construction project management, reducing delays and costs.

Data Collection

In this particular case study, previous similar building construction project data was reviewed to acquire the necessary data and information about the resources needed for the 3- bedroom construction project. This method in combination with past project records of cost estimates provided the required information. By collecting and analyzing the building resources data, it was possible to make informed decisions to optimize resource utilization.

After the review of templates, the tasks sheet was prepared together with their corresponding resources needed to complete them. This work was limited to the executing and control stages of the construction work and does not consist of the project initiation, planning and closing stages. Also, resources such as building materials were classified into categories. This was necessary because pieces of materials such as sand and granite could not be counted while some of the materials are also of lessor cost as compared to others.

List of Tasks and the Resources needed to Complete them

Table 1 shows the list of tasks and the resources required to complete tasks.

Table 1. List of Task and Resources Needed

Task	Resources Needed
Site preparation	6 laborers
	1 excavator and 1 grader
	Building materials: Shovels, pick axes and wheel barrows
	1 supervisor
	1 quality control engineer
	1 architect
Foundation	8 laborers including a steel bender
	1 concrete mixer
	Building Materials: Sand, silt, Laterite, Cement, Water, granite chips, iron rods, binding wires, wheel barrows, Shovels & pans.
	1 supervisor
	1 quality control engineer
	0 architect
Structural work	11 laborers including steel bender
	1 concrete mixer
	Building Materials: Sand, silt, Laterite, Cement, Water, granite chips, iron rods, binding wires, wheel barrows, Shovels & pans, wood
	1 building supervisor
	1 quality control engineer
	0 architect
Roofing and wood works	carpenters, 2 laborers
	machinery (generator machine) to tight screws
	Building Materials: Roofing sheets, Nails, wood, ladder,

	1 Supervisor
	1 Quality Control
	0 Architect
Electrical work	1 electrician and 2 apprentices
	1 voltmeter, 1 insulation tester
	Building Materials: Wiring conduit pipes, cement, wires, miniature circuit breaker, switches and sockets with screws, chisel and hammer,
	1 wiring supervisor
	1 quality control tester
	0 architect
Plumbing and tilling work	1 professional plumber, 2 plumbing apprentices, 2 tillers, 4 apprentices
	1 tiling Machinery
	Building Materials: Plumbing pipes, plumbers insulation tape with glue, Shower and accessories, toilet seat, kitchen and washroom sinks, water closets. Cement, tiles cement, water, tiles, sand.
	1 tile supervisor, 1 plumb supervisor
	1 quality control plumber, 1 quality control tiller
	0 architect

The resources in table 1 are then converted to their approximate numerical coded values and entered into a table to create a task – resource matrix as depicted by table 2. For instance, at each stage, every human involved in the project was counted as 1 unit. Cement was counted as a unit. Every

machine was also counted as a unit. Some materials of lessor cost were combined as a single unit.

Table 2 shows the types and quantities of materials needed for each stage of the construction process.

Table 2. Table of Task against Resource Quantities

	Resources					
	Labour	Machinery	Building Materials	Supervisor	Quality Control	Architect
Site Preparation	4	1	2	1	1	1
Foundattion work	8	1	8	1	1	0
Structural work	10	1	8	1	1	0
Roofing	4	0	5	1	1	0
Electrical work	3	0	6	1	1	0
Plumbing work	3	0	6	1	1	0
Tilling work	4	1	5	1	1	0
Painting work	3	o	3	1	1	0

The data in the form of an $n \times n$ square matrix was then created and further analyzed using eigen values and eigen vectors to determine the

critical resources necessary for the execution of the project tasks.

Data Analysis

In this part of the study, a 6×6 rectangular matrix was created. Since the main objective was to optimize the resources to the project task

$$\begin{bmatrix} 4-\lambda & 1 & 2 & 1 & 1 & 1 \\ 8 & 1-\lambda & 10 & 1 & 1 & 0 \\ 10 & 1 & 9-\lambda & 1 & 1 & 0 \\ 4 & 0 & 4 & 1-\lambda & 1 & 0 \\ 3 & 2 & 6 & 1 & 1-\lambda & 0 \\ 9 & 1 & 11 & 2 & 2 & 0-\lambda \end{bmatrix} = 0$$

Next, the eigen matrix was then decomposed to obtained the respective eigen values for $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ and λ_6 . However, the matrix was a 6×6 matrix and performing

by applying the eigen values and eigen vectors analysis to the matrix, the eigen values were entered into the matrix A and the characteristic equation created as follows.

manual calculation was complex. MATLAB program was then applied to estimate the eigen values.

```
% Define the matrix A
A = [4 1 2 1 1 1;
     8 1 10 1 1 0;
     10 1 9 1 1 0;
     4 0 4 1 1 0;
     3 2 6 1 1 0;
     9 1 11 2 2 0];
```

```
% Calculate eigenvalues and eigenvectors
[V,D] = eig(A);

% Display eigenvalues
eigenvalues = diag(D);

disp('Eigenvalues:');
disp(eigenvalues);

% Display eigenvectors
disp('Eigenvectors:');
disp(V);
```

By conducting analysis of the MATLAB code, it was possible to gain valuable insights of the eigen vectors and values with respect to the task- resource allocation matrix. This assisted in the resource allocation patterns, task groupings, and optimization opportunities.

Results

After creating a group using the task-resource data, simulation was performed using

MATLAB. This resulted in both the eigen values and the eigen vector values.

The eigenvalues of matrix A are:

Eigenvalues = $\lambda_1 = 23.4674, \lambda_2 = -0.4834 + 1.4954i, \lambda_3 = -0.4834 - 1.4954i, \lambda_4 = 0.9258, \lambda_5 = -0.2130, \lambda_6 = -0.2130$ and the eigen vectors of matrix A are:

Eigen Vectors, V =

0.2708	0.4082	0.4082	0.5774	0.5774	0.5774
0.5254	$0.4082 + 0.4082i$	$0.4082 - 0.4082i$	$-0.2887 + 0.5000i$	$-0.2887 - 0.5000i$	-0.2887
0.5000	0.4082	0.4082	-0.2887	-0.2887	-0.2887
0.2158	$0.4082 + 0.2041i$	$0.4082 - 0.2041i$	0.2887	0.2887	0.2887
0.3451	$0.2041 + 0.4082i$	$0.2041 - 0.4082i$	0.2887	0.2887	0.2887
0.4968	0.0000	0.0000	0.2887	0.2887	0.2887

The weighted sum of all the eigen vectors, denoted as S_v , is;

$$S_v = \sum v_i i = 1, 2, 3 \dots \dots .6$$

$$s_v = \begin{pmatrix} 2.819 \\ 0.4757 \\ 0.4503 \\ 1.8983 \\ 1.6194 \\ 1.3629 \end{pmatrix}$$

The oscillating eigenvalues and eigen vectors (complex with non-zero imaginary parts) in the task-resource matrix indicated that the resources have cyclical dependencies, where changes in one task/resource affects others in a cyclical manner which was shown throughout the construction project process. It also shows that there is a feedback loop in the system, a situation where outputs become inputs [10], causing oscillations and that there is a potential instability in the system, where small changes in resources can lead to amplified oscillations. These oscillatory eigen values and vectors created some project resource allocation challenges which was addressed by careful planning that investigated the task and resource - task dependencies to understand the pattern.

Discussion

The eigenvectors represent the directions of the new axes (principal components) that describe the variance within the data. The first Eigenvectors for $\lambda_1 = 23.4674$ is high for Foundation work (0.5254) and Structural work (0.5000). These tasks were critical for the commencement of the project. They were therefore, necessary for the whole building structure to be realized. Site preparation (0.2708), even though, recorded positive value, is important but not critical as it does not

consume major resources at the project inception. The second and third eigenvectors for ($\lambda_2 = -0.4834 + 1.4954i, \lambda_3 = -0.4834 - 1.4954i$) were all high except for plumping and tilling which did not used more resources as they occurred at the finishing phases of the construction. The fourth, fifth and sixth eigen vectors for ($\lambda_4 = 0.9258, \lambda_5 = -0.2130, \lambda_6 = -0.2130$) showed site preparation, electrical work, roofing, tilling and plumbing works recording positive values whiles foundation and structural work recorded negative values. These occurred due to the fact that foundation and structural work were completed and resources were diverted to the other critical areas. However, on the overall, the site preparation scored the highest vector value $s_v = 2,819$. The presence of an architect, which completed its matrix entries at the initial stage was enough to draw the conclusion that it is very important.

The simulation shows promising results. The method was limited to linear systems. This indicates that the resources in the construction project have linear dependencies. Based on this fact, it is recommended that this method of the study be applied in a very larger and a complex construction project setting.

Conflict of Interest

The author declares that there is no conflict of interest as the whole study was meant to learn and improve myself in both academia and the industry.

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